TOPICS IN COMPLEX ANALYSIS, HARMONIC ANALYSIS, AND COMPLEX GEOMETRY

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Overview

Generally speaking, I have broad interests in mathematical analysis, including **complex analysis (in one and in several variables), harmonic analysis, functional analysis, probability theory and stochastic processes, and dynamical systems**, and I would be happy to advise a PhD student who wishes to work in one or more of these subfields.

Here is a selection of some specific topics I have been interested in since 2020, with references listed at the end.

- **Stable polynomials and boundary behavior of rational functions in $\mathbb{C}^d$**:

  A polynomial $p \in \mathbb{C}[z_1, \dots, z_d]$ is said to be stable with respect to some open set $\Omega \subset \mathbb{C}^d$ if p has no zeros in Ω; p may well have zeros in the closure of Ω. Such polynomials arise naturally in several contexts. For instance, they are numerators of rational functions that are holomorphic in Ω. How can boundary properties of such q/p be related to the manner in which the zero set of p approaches $\partial\Omega$? Does the requirement that p be non-vanishing force p to have special algebraic properties, and can this be used to further probe rational functions having p as their denominators?
  - (with K. Bickel, G. Knese, J.E. Pascoe) *Local theory of stable polynomials and bounded rational functions of several variables*, https://arxiv.org/abs/2109.07507
  - (with K. Bickel and J.E. Pascoe) *Singularities of rational inner functions in higher dimensions*, Amer. J. Math. (144) 2022, 1115-1157.
- **Clark measures for holomorphic functions in several variables**:

  A bounded holomorphic function is called inner in $\Omega \subset \mathbb{C}^d$ if its boundary values (in a suitable sense) are unimodular. Given an inner function ϕ, we consider the expression
  $$\frac{1-|\phi(z)|^2}{|\alpha-\phi(z)|^2}, \quad z \in \Omega,$$
  and note that this is a positive pluriharmonic function. As such, it can (for reasonable Ω) be represented as the Poisson integral of a special type of positive measure μ_ϕ which is supported on part of, or all of, $\partial\Omega$. How does the structure of this measure reflect the nature of the inner function ϕ? How do operations of ϕ translate to modifications of μ_ϕ? What can we say about the structure of the space $L^2(\mu_\sigma)$.

- (with K. Bickel and J.A. Cima) *Clark measures for rational inner functions*, Michigan Math. J. (73) 2023, 1021-1057.
- (with J.T. Anderson, L. Bergvist, K. Bickel, and J.A. Cima) *Clark measures for rational inner functions II*, Ark. Mat., to appear (2024).

- **Multipliers on Hardy-Orlicz and other function spaces**:

  Suppose $\mathcal{X}$ is a function space on $\mathbb{T}$, the circle, or on $\mathbb{T}^d$, which is defined in terms of properties of Fourier or Taylor coefficients $\{\hat{f}_n\}$. A sequence $\{\lambda_n\}$ is said to be a multiplier from $\mathcal{X}$ into ℓ^p if we have $\|\{\hat{f}_n\lambda_n\}\|_{\ell^p} \leq C\|f\|_{\mathcal{X}}$ for the respective norms. Given a specific space $\mathcal{X}$ of interest, can one characterize multipliers in a concrete and useful way? Using such characterizations, is it possible to devise simple necessary conditions (decay, regularity, etc) that f needs to satisfy in order to belong to $\mathcal{X}$, whose initial norm need not be defined in a straight-forward manner?

  - (with O. Bakas, S. Rodriguez-Lopez, and S. Pott) *Multipliers for Hardy-Orlicz spaces and applications*, https://arxiv.org/abs/2306.09874

Prerequisites

I would expect a student who wishes to work with me to have a strong background in pure mathematics in general, and in complex/harmonic/functional analysis in particular. In addition, I view it as very desirable for students to be willing to learn about other fields in mathematics, including (but not limited to) algebraic geometry, differential geometry, dynamical systems, operator theory, etc.

Projects

In my view, it is best to settle on a first project that suits the strengths and interests of the particular student. The above can be seen as some indication of my recent interests and expertise and directions where I could likely propose concrete first problems, but I am quite open-minded regarding suggestions from the potential student. I believe that patience, persistence, and an interest in lucid exposition are crucial in order to achieve success as a mathematician, and are in many ways more important than apparent brilliance and speed.

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